

Comparison Between GVM and Wavelet GVM Model to Forecast Monthly Electricity Demand of Erbil Governorate

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Abstract—Collecting and getting electricity demand is one of the most important projects at any place and community collecting information about its consumption. As a result of not having a good strategy for generating and its consumption might be useless economically. There should be a good plan to develop generating electricity projects to decrease the load problem. The main purpose of this study is to estimate the Grey Verhulst model to realize the average demand for electricity per month. And knowing the rate of electricity consumption and appropriate treatments for this problem. All of the time typical technical series models are used for data to achieve and collect the results. There is a set of theories that can be performed for analysis in this case, grey system theory is one of those theories which is including a set of models like GM (1, 1), GVM, FGM and FRGM to allow expectations for some information in the future based on a set of original data that is going to be less and uncertain data. Forecasting is a branch of statistics that deals with parameters and (OLS) are one of the methods of guessing parameters in mathematical examples, this study implemented Grey Verhulst Model on the amount of demand for electricity for twelve months (Mar-2016 to Feb-2017).

Keywords— Grey Verhulst Modelling, Forecast, Haar Wavelet, White-box black-box models

I. INTRODUCTION

This Apart from with a standard fee, electricity utility users are billed according to units consumed measured in kilowatt-hours (kWh) and are obligated to pay monthly or seasonal fee. However, a demand fee is also charged on users using exorbitant electricity levels and this plays an instrumental role in explaining electricity demand (Singh et al., 2013). Assuming that one wants to determine the

maximum amount of water flowing in a stream in the following month, a flat board can be hanged and fastened in the stream so as to swing up when more water is flowing and determine a way of preventing it from swinging back down when the water level drops back down. This is vital and plays

an essential role in determining the highest water flow and hence, the board measures the stream's peak flow. Synonymous to the board example, measuring electricity utility involves using demand meters measuring electricity flowing similar to the manner board example measures flowing water. As a result, demand meters can record the highest current during a billing period. However, the measuring process slightly complicated because the meter records an average flow for every 15-minute interval. Thus, customers are billed for the highest average 15-minute flow during the billing period.

So how does the demand affect the customer's electricity bill? The demand fee constitutes a smaller portion of the entire bill when customers consume huge volumes of electrical energy and a huge portion of the electricity bill if customer consume a lot of electrical energy at a more or less constant rate throughout the month. Nonetheless, the total operational time of the pump does influence the energy charge portion of the electrical bill. Therefore, rational customers will avoid the pump operating for longer periods (less hours than necessary) so as to save money on the energy bill.

1.2 The aim of the study

This study aims to forecast the monthly average electricity demand in the next six months by using GVM.

2. Methodology

2.1 Introduction:

Since its inception by Deng Julong (1982), the Grey System Theory serves an essential role in estimating the behaviour of an uncertain system. The theory has been applauded for not requiring a limited amount of data (Liu, Guo & Dang, 1999) and avoiding defects associated with statistical and conventional methods (Chang et al., 1998). Hence, it has a recommendable ability to accurately determine and effectively monitor systematic operational behaviour. The notable feature of the Grey System Theory is that it uses colour to describe information availability. For instance, Liu, S., Yang, Xie & Forrest, 2016) outlined that the colour "white" is used for representing complete information and the colour "black"

denotes unknown information. Nonetheless, the Grey System Theory has proved effective to produce reliable findings when used in examining uncertain systems with insufficient data, discrete data and multi-data inputs.

Meanwhile, Yin (2013) highlights that, unlike Grey Forecasting Theory (GFT), accurate predictions and examinations of the estimated parameters can be established when the utilised historical data is normally distributed and contains numerous observations, especially when using traditional prediction methods. Kayacan, Ulutas and Kaynak (2010) expressed favour in supporting the GFT citing that it does not demand strict assumptions about the data set for it to yield accurate predictions. Consequently, using the GFT simplifies data collection procedures and aids in making timely predictions. According to Liu, Forrest and Yang (2011), though the GFT is distinctively different from Fuzzy Logic, it uses sequence operators based on information coverage to examine the law of the subject's motivation and focus on objects with vague internal meanings and definite external extensions.

Nonetheless, the Grey System Theory (GST) has also been effectively deployed in numerous prediction applications. As a result, it has increasingly been applied to yield effective results, especially when incomplete information and discrete data are involved, especially under considerable uncertainty (Liu, Forrest & Yang, 2011). Furthermore, Kayacan, Ulutas and Kaynak (2010) underscore that the GST yields desired results in educational measurement and information when traditional statistical methods are inapplicable in situations involving inadequate data. Thus, the GST plays an instrumental role in predicting problems though there are some few concerns about the predicted accuracy of grey models (Yin, 2013). Meanwhile, prediction is referring to the ability to determine future details using historical information (Yin, 2013). As such, several prediction methods demand numerous historical data and statistical methods and tools so as to examine any potential system's characteristics (Liu, Forrest, & Yang, 2011).

2.2. Data analysis procedures

Factor analysis was applied in determining which of the behavioral antecedents' constructs were related (Bandalos & Finney, 2018). As per Shrestha's (2021) guidelines, constructs with a factor loading of at least 0.70 were considered related and capable of providing a valid understanding of how the selected 9 dimensions influence the adoption of internet banking. This was followed estimation of a path analysis crucial in depicting connections between the adoption of internet banking and the selected behavioral antecedents (Barrett, 2007). Discriminant validity was carried out using the Average Variance explained (AVE) on the belief that discriminant validity is established when the variables' AVE exceeds 0.50 (Barrett, 2007). Convergent validity was tested using the Fornell-Lacker method. Concerning the reliability of the model variables, Cronbach's alpha, the composite reliability tests were applied, and a benchmark of 0.70 was used for assessing the constructs' reliability (Hooper, Coughlan & Mullen, 2008). The study further proceeded to examine the estimated SEM's fitness

and robustness using the Standardized Root Mean Square Residual (SRMR), Chi-square test, and Normed Fit Index (NFI), (Barrett, 2007; Hooper, Coughlan & Mullen, 2008).

2.3. White-box black-box models:

A white-box model, which is also known as the clear box or a glass box, is a system where all vital information is available (Kroll, 2000). In order to develop a white-box model, Henard et al. (2016) asserts that white-box model requires one to adhere to its governing principles such as those stipulated by the Newton equations, especially when modelling a physical process. However, due to the comprise complex nature of several processes and systems, white-box models have proved to be overly complex and thus making it practically impossible to produce within reasonable amount of time (Petsiuk, Das & Saenko, 2018). As a result, Pintelas, Livieris and Pintelas (2020) proposed that the most feasible approach is to initially commence by measuring the external influences inputs to the system and the system's behavior and then proceed to ascertain the mathematical connection linking them without establishing details about what transpires inside the system, and this is known as system identification. According to Kroll (2000), system identification comprises of two models and these are:

- 1) **The Grey box model:** This is a model that is constructed using experimental and system data irrespective of the inability to determine activities taking place inside the system (Kroll, 2000). Under such cases, system identification can be deployed to estimate such model through what is termed type of semi-physical modeling regardless of the fact that it has numerous unknown free parameters (Kristensen, Madsen & Jørgensen, 2004). A notable example of such a model is the Monod saturation for microbial growth comprising a simple hyperbolic model association between growth rate and substrate concentration. However, this is justifiable using molecules binding to a substrate without detailing about the nature of the molecules' binding.
- 2) **The Black box model:** Pintelas, Livieris and Pintelas (2020) assert that this is a system that is developed without any prior model nor priori information and several identification algorithms tend to assume this form. In practical terms, all existing systems are somewhere between white-b models and the black-box. As a result, this concept only serves as an intuitive guide. Meanwhile, uncertainty is a situation characterized by unknown and/or imperfect information (Petsiuk, Das & Saenko, 2018). Uncertainty is widely observable in numerous fields and sectors such as information science, metrology, engineering, psychology sociology, finance, economics, statistics, physics, philosophy and insurance. Events or activities such as indolence and/or as well as stochastic and/or partially observable environments are considered to be the main factors behind uncertainty (Henard et al., 2016). Thus, it

influences either unknown or existing events and plays a vital role in predicting future physical measurements and events.

2.4. Advantage of Grey models:

- Grey Prediction Model has the following advantages;
- It is applicable in forecasting situations such as competitive environments where decision-makers have only access to limited historical data.
- Does not require numerous discrete data to describe an unknown system.
- It can yield reliable results regardless of constraints posed by the availability of a limited number of observations.
- It is applicable in situations where users are uncertain whether the data is representative or when large samples are not available.
- It is applicable in early effective factor assessment.

2.5. Attributes of the Grey Model with a traditional forecasting model

Table 2.1 provides a comparative analysis of the Grey Prediction Model in relation to other traditional forecasting models as suggested by Chiang et al. (1998). As per Table 2.1, it is evidenced that this model only requires limited data, current and short-term so as to predict a given value.

Table 2.1: Attributes of a traditional forecasting model

Mathematical Model	Minimum Observed	Type of sample	Sample interval	Mathematical requirements
Simple exponential function	5 - 10	Interval	Short	Basic
Regression analysis	10 - 20	Trend	Short	Middle
Box-Jenkins Models	50	Interval	Long	Advanced
Neural Network Models	Large numbers	Interval or not	Short	Advanced
Grey Models	4	Interval	Long	Basic

2.6. Time series Data and use of Grey Model:

According to Kroll, (2000), time series refers to a collection of equally time interval-sampled data points. Hence, time series prediction is a process through which a system's future values are forecasted based using existing or historical. Under normal circumstances, accurate predictions can be made using a pre-defined mathematical model. However, certain forms of time series data are highly non-stationary stochastic and thus, making it practically challenging to apply artificial neural networks or conventional linear statistical methods and fit a model to them. As a result, Kristensen, Madsen & Jørgensen, (2004) suggested applying the prediction theory so as to such a problem. Meanwhile, the application of the GST an

interdisciplinary scientific field and famously proved to be effective in dealing with systems with partially unknown parameters (Yaqub, S. N., Aziz, 2022; Liu et al., 2016; Yin, 2013). Being superior to conventional statistical models, grey models do not demand several data when determining the behaviour of unknown systems.

2.7. Grey System Theory:

Deng's proposition to apply the GST was in strive of efforts to deal with a system's uncertainty in as much as information is concerned. Thus, any system lacking information, as behavior document, operation mechanism and structure messages is known as a grey system (Kayacan, Ulutas & Kaynak, 2010). The GST comprises five major parts encompassing the grey control, grey programming, grey decision, grey relation, and grey prediction (Yin (2013). Consequently, the grey model is the core of GST that gathers existing data and uses it to obtain internal regularity without applying any assumptions. The intention is to make forecasting vital for policymakers and decision makers in making future predictions.

2.8. Grey Verhulst Model (GVM)

Efforts to describe some increasing process with a saturation point(s) such as the "S" curve led the Germany biologist Verhulst to develop what is now known as the Grey Verhulst Model (GVM). As such, the GVM has been widely applied in several applications in describing living creatures and their individual growth as well as population increase (Zheng, Tong & Ma, 2020). The GVM aims to restrict a real system's entire development and is a special type of a model within the grey system.

For an initial sequence, $x^{(0)} = (x^{(0)}_{(1)}, x^{(0)}_{(2)}, \dots, x^{(0)}_{(n)})$ the initial sequence $x^{(0)}$ is used to construct the Verhulst model directly as follows:

$$\frac{dx^{(0)}(t)}{dt} + \alpha x^{(0)}_{(t)} = \mathbf{b}(x^{(0)}_{(t)}) \tag{2.1}$$

Here α presents the development coefficient and \mathbf{b} denotes the grey action quantity. The solution of the parameter vector $\alpha' = [\alpha, \mathbf{b}]^T$ can be obtained by using the least square method.

$$\alpha' = [\alpha, \mathbf{b}]^T = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{Y} \tag{2.2}$$

Where:

$$\mathbf{B} = \begin{bmatrix} -0.5(x^{(0)}_{(1)} + x^{(0)}_{(2)}) & (0.5(x^{(0)}_{(1)} + x^{(0)}_{(2)}))^2 \\ -0.5(x^{(0)}_{(2)} + x^{(0)}_{(3)}) & (0.5(x^{(0)}_{(2)} + x^{(0)}_{(3)}))^2 \\ \vdots & \vdots \\ -0.5(x^{(0)}_{(n-1)} + x^{(0)}_{(n)}) & (0.5(x^{(0)}_{(n-1)} + x^{(0)}_{(n)}))^2 \end{bmatrix}$$

$$\mathbf{Y} = [x^{(0)}_{(2)} - x^{(0)}_{(1)}, x^{(0)}_{(3)} - x^{(0)}_{(2)}, \dots, x^{(0)}_{(n)} - x^{(0)}_{(n-1)}]^T$$

$$\hat{x}^{(0)}_{(k+1)} = \frac{\alpha x^{(0)}_{(1)}}{bx^{(0)}_{(1)} + (\alpha - bx^{(0)}_{(1)})e^{\alpha k}} \quad k = 0, 1, 2, \dots, n. \tag{2.3}$$

Equation (2.3) obtained the result of $\hat{x}^{(0)}$.

2.8.1. Mean Absolute Percentage Error (MAPE)

In order to predict the accuracy of the forecasting model in the thesis, the Mean Absolute Percentage Error (MAPE) index was used to evaluate the performance and reliability of the forecasting technique. It is defined as follows:

$$MAPE = \frac{1}{n} \sum_{k=2}^n \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\% \tag{2.4}$$

Where $x^{(0)}(k)$: The actual value in time period k
 $\hat{x}^{(0)}(k)$: The forecast value in time period k
 And the grade of MAPE is divided into four grades:

Table (2.2) shows the grade of MAPE

Precision rank	MAPE
Highly accurate	mape \leq 0.01
Good	mape \leq 0.05
Reasonable	mape \leq 0.10
Inaccurate	mape \geq 0.10

The lower the MAPE, the higher precision the forecasting model can achieve. In general, the MAPE below 0.01 is an accurate model and the MAPE between 0.01 and 0.05 is a good model with acceptable accuracy.

2.8.2 Precision Rate (p)

Precision Rate, which measures the level of the closeness of the statement of forecast quantity and the actual value, p is defined as follows: Precision Rate (p) = 1-MAPE (2.5)

Table (2.3) shows the grade of the precision rate

Precision rank	Precision Rate
Highly accurate	$p \leq 0.99$
Good	$p \leq 0.95$
Reasonable	$p \leq 0.90$
Inaccurate	$p \geq 0.90$

The higher the precision rate, the higher precision the forecasting model can achieve. In general, a precision rate greater than 0.99 is an accurate model and the precision rate between 0.99 and 0.95 is a good model with acceptable accuracy.

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2.9. Haar Wavelet [8]

The simplest type of wavelet is the Haar wavelet. In their discrete form, these wavelets are linked with a mathematical process that is known as the Haar transform. This process plays the role of a prototype that facilitates other wavelet transforms. If we want to get a good understanding of the sophisticated wavelet transforms, we need to gather more information about the Haar transform. In short, this wavelet transform can be considered the most suitable choice regarding localized jumps and edge detection.

The numeric definition for the Haar scaling function is as follows:

$$\phi(x) = \begin{cases} 1, & \text{if } 0 \leq x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Whereas, the definition for the Haar mother wavelet is as follows:

$$\psi(x) = \phi(2x) - \phi(2x - 1) \tag{2.6}$$

$$\psi(x) = \begin{cases} 1, & 0 \leq x < \frac{1}{2} \\ -1, & \frac{1}{2} \leq x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

2.9.1 Haar Wavelet's properties [8]

- Any function can be used as the constant function's linear combination, $\psi(x), \psi(2x), \psi(2^2x), \dots, \psi(2^kx), \dots$ and their shifting functions.
- Another property is that any function can be used as the linear combination of $\phi(x), \phi(x2), \phi(2^2x), \dots, \phi(2^kx), \dots$ and associated shifting functions.
- Another property is that only Haar Wavelet can be compactly supported orthogonal and has symmetry
- The last property is that it makes use of a set of functions $\{2^{\frac{j}{2}} \phi(2^j x - k); k \in Z\}$ on orthonormal basis.

2.9.2 1-Level Haar Transform [8]

1st level Haar Transform for $f = (x_1, x_2, x_3, \dots, x_N)$ is given by:

$$f \xrightarrow{H1} (a^1 | d^1) \tag{2.7}$$

$$a^1 = \frac{x_1+x_2}{\sqrt{2}}, \frac{x_3+x_4}{\sqrt{2}}, \dots, \frac{x_{N-1}+x_N}{\sqrt{2}} \tag{2.8}$$

$$d^1 = \frac{x_1-x_2}{\sqrt{2}}, \frac{x_3-x_4}{\sqrt{2}}, \dots, \frac{x_{N-1}-x_N}{\sqrt{2}}$$

..... (2.1)

The list continues so for other levels.

2.9.3 Advantages of the Haar Wavelet Transform [8]

It is to be mentioned that the Haar Wavelet Transform offers multiple advantages. Some of the main advantages of the Haar Wavelet Transform include

- conceptually simple,
- fastest possible wavelet,
- fast processing speed,
- reversibility, without the edge effects that are a problem with other Wavelet transforms.
- increased memory efficiency, since it can be calculated in place without a temporary Array.

2.9.4 The Haar Transform Limitations [8]

As for the limitations of the Haar Transform, which can be a problem with some applications? In generating each of the averages for the next level and each set of coefficients, the Haar transform performs an average and difference on a pair of values. Not only this, when generating the averages to be used at the next level as well as for each set of coefficients, this transform performs concerning the pair of values in which the algorithm makes a shift over by two consecutive values for calculating the different level. Moreover, the Haar transform window is only wide by two elements so if in case a big change occurs from an even to an odd value, we cannot see the change in the high-frequency coefficients. Therefore, it can be said that for audio signal compressing and for noise removal, the Haar wavelet transform cannot be a viable choice.

3.1 Data Description:

The data were collected from the province of Hawler / Directorate of control and communication for electricity from the April 2021 to March 2022 in the average of monthly power energy (demand) as (12 consecutive Obs.) as at time series (T=1,2,...,12). The data is measured by kilowatt (kW), the data is shown in Table (3-1).

Table (3-1)

Shows the data collected of the study

T	Demand
1	102.49
2	104.11
3	107.34
4	103.32
5	94.14
6	95.75
7	97.96
8	101.71
9	102.36
10	101.12
11	109.36
12	108.6

Table (3-1) represents the data collection of the study during twelve periods of time, the data is the monthly average.

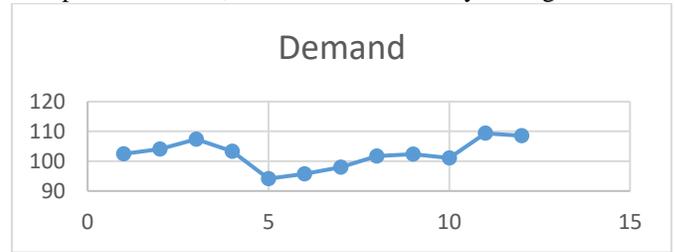


Figure (3-1) Shows the average monthly electricity demand

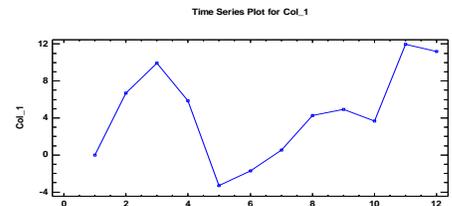
Figure (3-1) shows the average monthly electricity demand as it is shown the electricity demand from the first three months is increasing that started from (102) and reaching (107) but in months four and five the electricity demand is decreasing.

3.2 Model Specifications

The data that has been collected as described in previous sections are used to perform grey model analysis.

3.3.Grey Verhulst Model (GVM) Fitting

The



procedure of Grey model GVM can be shown in steps as follow:

Depending on Eq (2.2), by using the OLS method the GVM model values of the essential terms are presented as:

$$B = \begin{pmatrix} -103.3 & 10670.89 \\ -105.725 & 11177.78 \\ -105.33 & 11094.41 \\ -98.73 & 9747.613 \\ -94.945 & 9014.553 \\ -96.855 & 9380.891 \\ -99.835 & 9967.027 \\ -102.035 & 10411.14 \\ -101.74 & 10351.03 \\ -105.24 & 11075.46 \\ -108.98 & 11876.64 \end{pmatrix}$$

$$B^T B = \begin{pmatrix} 114767.4 & 1.2E+07 \\ 1.2E+07 & 1.2E+09 \end{pmatrix}$$

$$(B^T B)^{-1} = \begin{pmatrix} 0.005758 & 5.62E-05 \\ 5.62E-05 & 5.48E-07 \end{pmatrix}$$

$$B \cdot Y = \begin{pmatrix} -644.88 \\ 68097.49 \end{pmatrix}$$

$$\hat{a} = \begin{matrix} a & = & 0.11089 \\ b & = & 0.001138 \end{matrix}$$

$$Y = \begin{pmatrix} 1.62 \\ 3.23 \\ -4.02 \\ -9.18 \\ 1.61 \\ 2.21 \\ 3.75 \\ 0.65 \\ -1.24 \\ 8.24 \\ -0.76 \end{pmatrix}$$

3.4 Goodness of fit of GVM

GVM is a time series model then the goodness of fit of the model should be tested by using a Box-price test that depends on the ACF of the residuals of the GVM the results were as follow:

Figure (3-2)

Shows the scatter plot of the residual series of the GVM model

Box-Pierce Test

Test based on first 4 autocorrelations large sample test statistic = 3.17316

P-value = 0.529278

The test is based on the sum of squares of the first 24 autocorrelation coefficients. Since the P-value for this test is greater than or equal to 0.05, we cannot reject the hypothesis that the series is random at the 95.0% or higher confidence level, it means that the residuals of the GVM model are distributed normally this is an indication of the goodness of fit for the GVM model.

Table (3-2)

Represents the accuracy of GVM and Wavelet GVM

Test	GVM Accuracy	Wavelet GVM Accuracy
MAPE	4.940%	4.0913%
Precision Rete	95.060%	95.909%

Accuracy

From the above table value of MAPE of both is less than 5% which means the postulated models are accurate, also the value of precision rate is greater than 95% which means the postulated models are highly accurate.

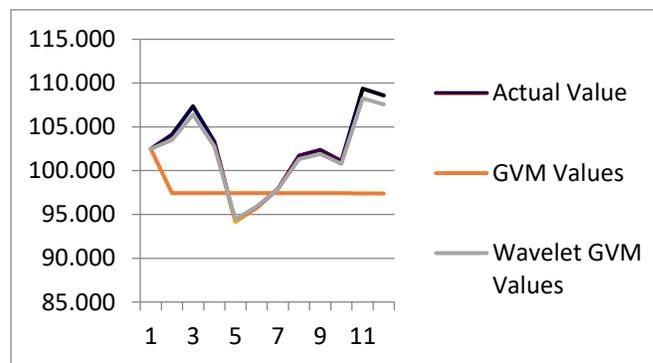
3.5 Estimating Prediction Values

After testing the goodness of fit of GVM we should calculate the predicted values of electricity demand depending on eq. (2.3) and calculating the residuals of the GVM model and transforming it into a haar-wavelet then computing, as it is shown below in table (3-2).

Table (3-3)

Represents the predicted values of electricity demand

T	Actual Value	GVM Values	GVM Residuals	Wavelet GVM Values	Wavelet GVM Residuals
1	102.490	102.490	0.000	102.490	0.000
2	104.110	97.433	6.677	103.489	0.621
3	107.340	97.427	9.913	106.418	0.922
4	103.320	97.432	5.888	102.773	0.547
5	94.140	97.449	-3.309	94.448	-0.308
6	95.750	97.447	-1.697	95.908	-0.158
7	97.960	97.442	0.518	97.912	0.048
8	101.710	97.431	4.279	101.312	0.398
9	102.360	97.428	4.932	101.901	0.459
10	101.120	97.430	3.690	100.777	0.343
11	109.360	97.397	11.963	108.248	1.112
12	108.600	97.395	11.205	107.558	1.042



From the above figure, it is clear that the predicted values of the electricity demand of Erbil are approximately between 102.49 and 97.395 kW.

4 .Conclusions

- 1- From the results, it can be concluded that the Wavelet GVM is better than GVM to represent the behavior of precipitation rate in the Erbil governorate.
- 2- Erbil governorate electricity demand declines towards the last two months of the year as an indication of companies and individuals reducing work-related activities in preparation of the new year.

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