# Using Queuing Theory to Choosing the Best Model for the Number of Recent Births and Deaths Data in Maternity Hospital - Erbil

Drakhshan Jalal Hassan [1] Feink Mohammed Omer [2] Esraa Awni Haydier [3]

[1,3], Statistics and Informatics Department, Erbil College of Administration and Economics- Salahaddin University, Kurdistan Region – Iraq

[2] Statistics and Informatics Department Erbil College of Administration and Economics - Salahaddin University, Kurdistan Region – Iraq

***Abstract*—In this study, the researcher endeavors to reduce the waiting time for beneficiaries (customers) in order to ensure beneficiary satisfaction and prevent them from leaving without receiving the service, thus solidifying their conviction to return to this service in the future. Quantitative methods and operations research are scientific approaches that assist decision-makers in making precise and objective decisions due to their capability to model and simplify complex problems. The issue of forming waiting queues is among the most prominent challenges that management in service organizations faces.**

**For the purpose of implementing the queuing model, data regarding the arrival of beneficiaries at the service station in the maternity hospital in Erbil Governorate were collected over a period of two years (2021 and 2022) by month. The data of arrivals and departures were tested using the (Goodness of Fit) test, revealing that the data follows a Poisson distribution. To apply queuing theory models, the selected model was the multi-service center model, given that the system consists of (3) delivery rooms in the hospital, and this specific room is considered the service hub.**

**Finally, a set of conclusions were reached, the most important of which is the probability that service centers (rooms) are occupied, which is a certain probability (1), The probability of no room being available in the hospital is equal to (0.36), indicating a weak probability concerning the studied phenomenon and the average number of newborn infants in the hospital (system) is (1).**

**Keywords: The Concept of Queue Models, Reasons for the Appearance of Queue Models, Queue Models.**

## I. INTRODUCTION

Queue models are commonly used in everyday life when customers ask to wait in queues for a service, such as: queue in supermarket, waiting for food in the restaurant, in the doctor's waiting room, at the bank, cars waiting at the traffic light, etc. Queue models are part of queue theory because the model will be built so that queue lengths and waiting time can be estimated. Therefore, there are many areas to work on using queue theory to improve the design of processing systems such as medicine, commerce, transportation fields, etc. (**Kalashnikov - 1994**)

Row theory is a useful tool for making decisions about determining hospital needs and resources such as the number of doctors per unit, the number of medical instruments, the number of waiting rooms, etc. It has been successfully used in the study of healthcare service problems. Therefore, the research seeks to build a perception of the theory of queue models and its application to the maternity hospital in Erbil in general with the aim of improving the level of performance in the operating hospital at a level that meets the needs of the patient. (**Taha - 2007**)

This work includes the theoretical aspect of row theory, which includes definitions and basic terms related to research and applied side of the research, where it is a description of data and the results in the application of row theory

### 2: Aims of the research

This research aims at several points related to patients coming to the maternity hospital in Erbil Governorate:

- Calculate average patient reach rate and service rate so as to reduce patient waiting time and make them more satisfied with the system.
- Calculate the average number of patients in the system and in the queue for the form.
- Choosing the optimal models for the maternity unit in the hospital and the most satisfactory for patients and hospital management.

### 3: The Concept of Queue Models

The historical origins of the idea of queue models go back to the year (1909) at the hands of the Danish engineer (Erlang)


Regular research paper: Published 18th May 2024

when he conducted his first experiments to address the problem of frequent phone calls and find solutions to it, as he found that phone call seekers are subject to delays due to the inability of female workers to fulfill orders at the required speed (**Guo & Wong - 2017**). Where the problem was addressed by calculating waiting times and service time. Thus, the concept of queue models is one of the oldest methods of management science, which has been widely used in solving many practical problems facing organizations, as the use of queue models theory was not limited to providing services and industrial operations only, but also extended in depth to military uses since World War II (**Allen - 1990**). Where queue models are considered one of the theories that have importance in human life, as they deal with waiting times for customers, and on the other hand, they are important for operations managers in dealing with bottlenecks that occur at work and ways to face the accumulation of demand and achieve a balance between the ability to provide producers and maintain customer satisfaction.

The theory of queue models is a mathematical study of what is called waiting or queue models , and this phenomenon is common in daily life, such as repair shops, gas stations, train stations, airports, and other common daily examples, where a queue models arises when there is a service station or several service stations in front of which a number of units of order to get The service must wait, due to the inability of that station or stations to accommodate those units, and then provide the service as soon as it arrives at the station. (**Bhat - 2015**)

Also, the theory of Queue models is a study of the processes that are characterized by random access, and this means that the access of units to the service channel is at random intervals, as well as the service is also a random process, and independence is one of the basic assumptions in building a line model wait.

**4: Reasons for the Appearance of Queue models**

Queue models appear significantly in developing countries, especially in service organizations it decreases in developed countries until it almost does not appear, and the appearance of queue models. is due to many reasons, the most important of them:

**4.1. Availability of the service system:**

Organizations in developed countries focus on building rules and regulations that control behavior and directing it to achieve the goal, as the system follows a number of rules in the field of service provision, the most important of which are: (**Cooper - 1981**)

**A-** What arrives at the service center is first-come-first-served, such as customer service, aircraft, and ships.

**B-** With what arrives at the service center last served first, such as the process of storing goods in warehouses.

**C-** Priority for certain groups, such as (boarding of the disabled on means of transportation or in the case of service provision.

**4.2. Behavior of service seekers**

The behavior of service seekers has a significant impact on the formation of queue models, as behavior is affected by the availability of a service system that guarantees discipline and commitment. Among the aspects of behavior that have an impact on the waiting line are: (**Cooper - 1981**)

**A-** The service applicants refused to stand in the waiting line.

**b-** Transfer of service seekers from one line to another.

C- Focusing service seekers on a limited time.

**5: Assumptions of Queue models theory**

Queue theory is to build a mathematical model of different types of queuing systems so that it can predict how the system will be. Queue system or waiting line can be better described as a line consisting of incoming clients or items that are formed in front of servers or service facilities in order to obtain the expected services. Queue lines theory is based on the following assumptions: (**Sundarapandian – 2009**)

1- Arrival service is provided on a first-come, first-served basis

2- Every customer is waiting to be served regardless of the length of the line.

3- Arrival is independent of previous arrivals, but the average number of arrivals does not change over time.

4- Service times also vary from one customer to another and are independent of each other, but their average is known.

5- Service times occur according to a negative exponential probability distribution.

6- The average service time is greater than the average arrival time.

**6: Queue Models**

There are many models of queue models used in the field of operations management, we will look at them three models, which are most prevalent: (**Gross, et. - 2008**)

**6.1: The Simple Model (M/M/1) :(GD/N/∞)**

It is the one-channel model for service provision, and it is assumed that conditions are met in this system, namely:

- Random access to customers follows the Poisson distribution with a rate of $(\lambda)$ per time unit.
- The service time follows an exponential distribution at a rate of $(\mu)$ per time unit.

The effectiveness measures of this model can be determined by calculating the following indicators:

- The service provider may be busy it is: $p = \frac{\lambda}{\mu}$ ... (1)
- Probability distribution of the service Possible lack of any unit in the system: $p_0 = 1 - \frac{\lambda}{\mu}$ ... (2)
- Probability of having one customer in the

system: $p_1 = \left(\frac{\lambda}{\mu}\right) p_0$ ... (3)

- The probability that there are n clients in the system: $p_n = \left(\frac{\lambda}{\mu}\right)^n p_0$ ... (4)
- Average number of clients in the system: $L = \left(\frac{\lambda}{\mu}\right) \Big/ 1 - \left(\frac{\lambda}{\mu}\right)$ ... (5)
- Average number of customers in line: $L_q = \left(\frac{\lambda}{\mu}\right)^2 \Big/ 1 - \left(\frac{\lambda}{\mu}\right)$ ... (6)
- The average time spent for a single customer in the system: $W = 1/(\mu - \lambda)$ ... (7)
- Average time spent in line for one customer: $W_q = \left(\lambda/\mu\right)\left[1/(\mu - \lambda)\right]$ ... (8)

**6.2: Single-Channel Service Model and Limited Queue Length**

What distinguishes this model from its predecessor is that the number of customers in the system is limited or does not exceed a specific number of them, and the conditions of this model are the same as the conditions of the previous model, in addition to the condition that the capacity of the system is limited to a certain number of customers, let it be (M)

Thus, the effectiveness measure of this model can be determined by calculating the following indicators:

- The service provider may be busy it is: $p = \left(\frac{\lambda}{\mu}\right)^{M+1}$ ... (9)
- Probability distribution of the service Possible lack of any unit in the system: $p_0 = 1 - \left(\frac{\lambda}{\mu}\right)^{M+1}$ ... (10)
- The probability that there are M clients in the system: $p_M = \left(\frac{\lambda}{\mu}\right)^M p_0$ ... (11)
- Average number of clients in the system: $L = \left[\left(\frac{\lambda}{\mu}\right) \Big/ 1 - \lambda\mu\right] - \left[(M+1)P \big/ P_0\right]$ ... (12)
- Average number of customers in line: $L_q = L - (1 - P_0)$ ... (13)
- The average time spent for a single customer in the system: $W = L \big/ \lambda(1 - P_M)$ ... (14)
- Average time spent in line for one customer: $W_q = L_q \big/ \lambda(1 - P_M)$ ... (15)

**6.3: Multiple Service Centers Model**

Under this system, there is more than one center to provide service, where customers stand in a row and then head to the center. The service center available to receive the service from.

This model assumes the same conditions as the simple model, except that due to the multiplicity of service centers, the condition of the access rate is less than the service rate becomes ($S\mu > \lambda$) where (S) refers to the number of service centers.

In this way, the effectiveness measures of this model can be determined by calculating the following indicators:

- Possibly busy service centers: $p = \frac{\lambda}{S\mu}$ ... (16)
- Possible disruption of facilities or service: $p_0 = \left[\sum_{n=0}^{S-1} \frac{\rho^n}{n!} + \frac{\rho^S}{S!\left(1-\frac{\rho}{S}\right)}\right]^{-1}$ ... (17)
- The probability that there are (n) customers in the system and there are two conditions

  If it was: $(S \geq n)$ ⟶ $p_n = \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} P_0$ ... (18)

  If it was: $(S \leq n)$ ⟶ $p_n = \frac{\left(\frac{\lambda}{\mu}\right)^n}{S!} S^{n-S} P_0$ ... (19)
- The average number of clients in the system: $L = L_q + \left(\lambda/\mu\right)$ ... (20)
- Average number of customers in line: $L_q = P_0 \left(\lambda/\mu\right)^S \left(P/S!\right)(1 - P)^2$ ... (21)
- The average time spent for a single customer in the system: $W = W_q + \left(1/\mu\right)$ ... (22)

Average time spent in line for one customer: $W_q = \left(L_q/\mu\right)$ ... (23)

**7: Data Collection**

In this study we have dealt with the number of child who are recent births and deaths in the families, where the data for our study was collected at the maternity hospital in Erbil Governorate for the year 2021 and 2022 according by months, our attention on the available data define in the following tables: -

**Table (1)**: Represents the number of recent deaths for 2021and 2022 year by months

| Months / Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2021 | 26 | 25 | 21 | 17 | 7 | 5 | 5 | 9 | 14 | 11 | 8 | 8 |
| 2022 | 12 | 7 | 11 | 0 | 8 | 5 | 16 | 14 | 4 | 18 | 10 | 10 |

**Table (2)**: Represents the number of arrival recent births for 2021and 2022 year by months

| | Months / Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Arrival | 2021 | 1775 | 1675 | 1726 | 1554 | 1631 | 1620 | 1992 | 1911 | 1951 | 2031 | 2014 | 1051 |
| | 2022 | 1963 | 1725 | 1898 | 1603 | 1658 | 1696 | 1831 | 1817 | 1844 | 1896 | 1785 | 1792 |

**Table (3)**: Represents the number of departure recent births for 2021and 2022 year by months

| | Months / Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Departure | 2021 | 1749 | 1647 | 1705 | 1537 | 1624 | 1615 | 1917 | 1902 | 1937 | 2025 | 2006 | 2043 |
| | 2022 | 1951 | 1718 | 1887 | 1703 | 1650 | 1691 | 1815 | 1803 | 1840 | 1878 | 1775 | 1777 |

**8: Application of Queue Theory to Recent Birth Data**

In this section we applied one of the models of row theory (queue theory) on the data where were collected at the maternity hospital in Erbil Governorate, for this purpose we assume the number of births of children in the hospital delivery as arrivals and the children who are discharged from the hospital delivery alive and healthy are considered departure and for the row theory applied the multiple service center model, because the system consists of (3) delivery hall in the hospital and considering this hall is the caregiver.

**8.1: Test the distribution of the number of arrivals and departures from recent births**

**- Testing the distribution of the number of arrivals from recent births**

$H_0$: Arrivals ~ Poisson ($\lambda$ =1765.25).

$H_1$: Arrivals ≁ Poisson ($\lambda$ =1765.25).

Based on the good ness of fit tests, there is no evidence to reject the null hypothesis which state that the distribution of recent births arrival is behaving as a Poisson distribution with a mean equals to ($\lambda$ =1765.25) births per month because (Critical value = 0.1445> $\alpha = 0.05$).

**- Testing the distribution of the number of departures from recent births**

Regarding the hypothesis for the distribution of recent births departure, it is as follows:

$H_0$: Departure ~ Poisson ($\mu$ =1753.9583).

$H_1$: Departure ≁ Poisson ($\mu$ =1753.9583).

Also based on the good ness of fit tests, there is no evidence to reject the null hypothesis which state that the distribution of recent births departures is behaving as a Poisson distribution with a mean equals to ($\mu$ =1753.9583) births per month because (Critical value = 0.2799 > $\alpha = 0.05$).

There for the model ($GD/\infty/\infty$) ($M/M$/S) can be applied where knowing that (S) represents the number of rooms in the delivery hospital, in order to find the average number of recent births in the hospital and to find the probability that the hospital is occupied by a certain number of patients and other metrics. The basic condition for the application of this system are:

$$\lambda < S\mu$$
$$1765.25 < (1753.9583)(3)$$
$$1765.25 < 5261.8749$$

- Service centers (halls) may be occupied

$$\rho = \frac{\lambda}{\mu} = \frac{1765.25}{1753.9583} = 1$$

- The possibility of not having any room in the hospital

$$p_0 = \left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!\left(1-\frac{\rho}{s}\right)}\right]^{-1}$$

$$p_0 = \left[\frac{(1.0064)^0}{0!} + \frac{(1.0064)^1}{1!} + \frac{(1.0064)^2}{2!} + \frac{(1.0064)^3}{3!\left(1-\frac{1.0064}{3}\right)}\right]^{-1} = 0.3612$$

- Average number of babies waiting for birth in queue

$$L_q = \left(\frac{\rho^s}{s!}\right) p_0 \left(\frac{\rho}{s}\right)\left[\frac{1}{\left(1-\frac{\rho}{s}\right)^2}\right]$$

$$L_q = \left(\frac{(1.0064)^3}{3!}\right)(0.3612)\left(\frac{1.0064}{3}\right)\left[\frac{1}{\left(1-\frac{1.0064}{3}\right)^2}\right] = 0.0466$$

- The average number of newborns in the system

$$L_s = L_q + \rho$$

$$L_s = 0.0466 + 1.0064 = 1.053$$

The average number of newborns in the system is about one child at any given time.

- Average time elapsed for one newborn in queue

$$W_q = \frac{L_q}{\mu} = \frac{0.0466}{1753.9583} = 0.000027$$

- Average elapsed time for one newborn baby in the system

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = 0.000027 + \frac{1}{1753.9583} = 0.0006$$

- The probability of having n children in the system, and there are two conditions:

If : $n \leq 3$ then $p_n = \left(\frac{\rho^n}{n!}\right) p_0$

$$p_3 = \left[\frac{(1.0064)^3}{3!}\right](0.3612) \quad ; \quad n = 3$$

$$p_3 = 0.0614$$

$$p_2 = \left[\frac{(1.0064)^2}{2!}\right](0.3612) \quad ; \quad n = 2$$

$$p_2 = 0.1829$$

$$p_1 = \left[\frac{(1.0064)^1}{1!}\right](0.3612) \quad ; \quad n = 1$$

$$p_1 = 0.3635$$

If : $n \geq 3$ then $p_n = \left(\frac{\rho^n}{s!}\right) S^{n-s} p_0$

$$p_3 = \left[\frac{(1.0064)^3}{3!\ (3)^{3-3}}\right](0.3612) \quad n = 3$$

$$p_3 = 0.0614$$

$$p_4 = \left[\frac{(1.0064)^4}{3!\ (3)^{4-3}}\right](0.3612) \quad n = 4$$

$$p_4 = 0.0206$$

$$p_5 = \left[\frac{(1.0064)^5}{3!\ (3)^{5-3}}\right](0.3612) \quad n = 5$$

$$p_5 = 0.0069$$

**9: Conclusions:**

According to the results from the practical part we reach to the following conclusions: -

1- By describing the data on births and deaths:

- The highest number of recent births for the year 2021 was in the month of 12, where their number reached (2043).
- The highest number of births for the year 2022 was in 1 month, where their number reached (1951).
- The highest number of recent deaths for the year 2021 was in 1 month, where their number reached (26).
- The highest number of deaths for the year 2022 was in the month of 7, where their number reached (16).
- The highest number of arrivals from recent births was in the month of 10 for the year 2021, where their number reached (2031) expatriates.
- The highest number of hospital discharges from recent births was in the month of 10 for the year 2021, where their number reached (2020) departures.

2- Through the analysis of birth data, it was found that the distribution of the number of arrivals from recent births is distributed by Poisson at a rate of ($\lambda$ = 1765.25).

3- It was also found that the distribution of the number of hospital discharges from recent births is distributed by Poisson at a rate of ($\mu$ = 1753.9583).

4- Through the application of the multiple system of row theory, it was found that:

- The possibility that service centers (halls) are occupied is a certain possibility (1).
- The probability of not having any room in the hospital is equal to (0.36), which is a low probability for the studied phenomenon.
- The average number of children waiting in line waiting for birth is (0.05), i.e. there are no children in the waiting line.
- The average number of newborns in the hospital (system) is (1).
- The average elapsed time for one newborn in the queue is equal to (0.000027) and this is an indication that there is no child in the queue.
- The average elapsed time for one newborn child in the hospital is equal to (0.0006).
- If $n \leq 3$ The likelihood of children in the system

increases.

- If $n \geq 3$ The likelihood of children being in the system decreases.

## Reference

[1] Allen, A. O. (1990) “Probability, Statistics, and Queueing Theory: With Computer Science Applications”; 2nd Edition; Academic Press.

[2] Bhat, U. N. (2015) “An Introduction to Queueing Theory: Modeling and Analysis in Applications”; 2nd Edition; Boston, MA: Birkhauser.

[3] Cooper, R. B. (1981) “Introduction to Queuing Theory”; 2nd Edition; Elsevier North Holland, Inc.

[4] Gross, Donald; Shortle, John F.; Thompson, James M.; Harris, Carl M. (2008) “Fundamentals of Queuing Theory”; 4th Edition, John Wiley & Sons, Inc. Hoboken, New Jersey.

[5] Guo, X., Lai, T. L., Shek, H., & Wong, S. P.-S. (2017) “Quantitative Trading: Algorithms, Analytics, Data, Models, Optimization”; CRC.

[6] Kalashnikov, Vladimir.V. (1994) “Mathematical Methods in Queueing Theory”; Springer Science+ Media Dordrecht.

[7] Sundarapandian, V. (2009) “Probability, Statistics and Queueing Theory”; PHI Learning.

[8] Taha, H. A. (2007) “Operations Research: An Introduction”; 8th Edition; New Jersy: Pearson Prentice Hall.