Forecasting For Silver Closing Price and Modifying Predictions by Using Wavelet Transformation

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***Abstract*— Silver is a precious metal and the spot price not only reflects the current supply and demand condition but it also reflects investors' expectations of future inflation and other general business/economic conditions. Therefore, the main objective of this study is to forecasting yearly silver closing price and modifying the prediction by using wavelet transformation. This study aims to analyze the time series of yearly silver closing price for the period between (1969 to 2022) using time series analysis which is (Box-Jenkins) method for the accuracy and flexibility it has in addition modifying the prediction by using wavelet transformation. In this study. The study found that the fit and efficient model according to smallest measurements (RMSE, MAPE, MAE, and ME) is the ARIMA(2, 2, 1) model. According to the results of ARIMA(2, 2, 1), the amounts of yearly silver closing price have been modified by wavelet transformation which has smallest RMSE and ME when compared with the original ARIMA(2, 2, 1).**

***Keywords*—Time Series Analysis, ARIMA model, Forecasting, wavelet transformation**

1. Introduction

Silver was used as an investment like other precious metals. It has been regarded as a form of money and store of value for more than 4,000 years, although it lost its role as legal tender in developed countries when the use of the silver standard came to a final end in 1935. Some countries mint bullion and collector coins, however, such as the American Silver Eagle with nominal face values. In addition, the statistical tools could be analyzed these problems especially when time is a significant factor in them. Time series analysis is one of the powerful statistical tools that is used to forecasting yearly silver closing price which are causes of changing the economic situation. In this paper, we use autoregressive moving average (ARMA) time-series models and modifying model prediction using wavelet transformation. The AR and ARMA models are very well-known statistical methods for the analysis of stochastic processes in many diverse fields such as spectral estimation, time series forecasting and prediction, and biomedical engineering. The ARMA model is popular parametric approach, which uses the input and output signal to model the dynamics of

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physiological systems. In addition, for short-term prediction and forecasting, autoregressive integrative moving average (ARIMA), one variant of the ARMA model, provides more accurate results.

There are several studies that were used the ARIMA method, such as Biljana Petrevska and Goce Delcev (25 Jul 2016) they used autoregressive integrated moving average model to forecast the number of international tourism demand focusing on the case of F.Y.R. Macedonia, for this purpose, the Box–Jenkins methodology have been applied and several alternative specifications were tested in the modeling of original time series and international tourist arrivals recorded in the period 1956–2013. Through the results of standard indicators for accuracy testing, the best model was ARIMA(1,1,1) for forecasting, then according to the forecasted values the researchers found out that the number of international tourism will increase by 13.9% in 2018. empirically developed a univariate autoregressive moving average model for Nigerian inflation and analyzed their forecasting performance for data between 1982- 2010. The study showed that ARIMA (2,2,3) tracked the actual inflation appropriately. The conclusion drawn was that Nigerian inflation is largely expectations-driven. In addition, it showed that ARIMA models can explain Nigeria inflation dynamics successfully and help to predict future prices. In a study to forecast Bangladesh's inflation, applied a Box Jenkins ARIMA time series model. One year forecasting was done for consumer price index of Bangladesh using a structure for ARIMA forecasting model where a time series was expressed in terms of past values of itself plus current and lagged values of a 'white noise' error term were drawn up. Validity of the model was tested using standard statistical techniques and the best model was proposed on the basis of various diagnostic and selection & evaluation criteria. The study found many disadvantages of ARIMA model as it neglected the inclusion of explanatory variables and conducted the forecasts only on past values of the dependent variable in combination with present and past moving average terms. So, incorporating the judgmental elements with the selected ARIMA model can enhance the predictability of model for forecasting consumer price index of Kenya and better assist the policymakers. To the best knowledge of the researchers, no current literature on using wavelet ARIMA model to forecast of silver closing price, and this concept is our contribution in the current study. So that the objective of the study is to forecasting yearly silver closing price and modifying the prediction by using wavelet transformation.

So, the next section provides a brief overview of framework applying the Autoregressive Moving Average and wavelet transformation. In section 3, present the data and derive the time series models utilized in the analysis from the theoretical framework. The conclusions and further discussion of the study results are examined in section 4.

2. Materials and Methods

2.1 Time series

A time series is a sequence of data variables, which is consisting of successive observations on a quantifiable variable(s), that is making an over a time interval Usually, the observations are chronological and taken at regular intervals (days, months, years). Time series data are also often seen naturally in many field areas including; (Economics, Finance, Environmental, and Medicine) Time series can be represented as a set of observations X_t, each one being recorded at a specific time t and written as:

$\{X_1, X_2, \dots, X_t\}$ or $\{X_T\}$, where T = 1, 2,...t

and X_t is the value of X at time t, then the goal is to create a model of the form:

$$X_t = f(X_{t-1}, X_{t-2}, \dots, X_{t-n}) + e_t \quad \text{………………. (2.1)}$$

Where X_{t-1} is X_t variable for values of lag 1 that is the previous observations value, X_{t-2} is the Xt variable for values of lag 2 means two observations value ago, etc., and it represents noise value which doesn't follow a pattern of forecasting. The X_t value is usually highly correlated with X_t-cycle if a time series is following a pattern repeating, where the cycle was an observations number in a regular cycle.

2.2 Time series Analysis

Time series data occurrences are becoming extremely valuable to the operations and development of modern organizations. Financial institutions. Likewise, public and private institutions are using time series data to manage and project the loads on the networks. More and more time series are used in this type of investigation and hundreds of thousands of time series that contain valuable economic and financial information are nowadays available both on and off-line.

Time series analysis accounts for the fact that data points taken over time may have an internal

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structure (such as autocorrelation, trend or seasonal variation) that should be accounted for. As defined earlier, time series analysis comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data. It involves the use of techniques for drawing inferences from time series data. Note however that one other main purpose for analyzing time series is forecasting. Forecasting is the application of a model to predict future values based on previously observed time series values.

2.3 Stationary and Non-stationary Series:

Stationary series vary around a constant mean level, neither decreasing nor increasing systematically over time, with constant variance. Non-stationary series have systematic trends, such as linear, quadratic, and so on. A non-stationary series that can be made stationary by differencing is called "non-stationary in the homogenous sense." Stationarity is used as a tool in time series analysis, where the raw data are often transformed to become stationary. For example, economic data are often seasonal or dependent on a non-stationary price level. Using non-stationary time series produces unreliable and spurious results and leads to poor understanding and forecasting. The solution to the problem is to transform the time series data so that it becomes stationary. If the non-stationary process is a random walk with or without a drift, it is transformed to stationary process by differencing. Differencing the scores is the easiest way to make a non-stationary mean stationary (flat). The number of times you have to difference the scores to make the process stationary determines the value of d. If d=0, the model is already stationary and has no trend. When the series is differenced once, d=1 and linear trend is removed. When the difference is then differenced, d=2 and both linear and quadratic trend are removed. For non-stationary series, d values of 1 or 2 are usually adequate to make the mean stationary. If the time series data analysed exhibits a deterministic trend, the spurious results can be avoided by detrending. Sometimes the non-stationary series may combine a stochastic and deterministic trend at the same time and to avoid obtaining misleading results both differencing and detrending should be applied, as differencing will remove the trend in the variance and detrending will remove the deterministic trend.

A stationary process has the property that the mean, variance and autocorrelation structure do not change over time. Stationarity can be defined in precise mathematical terms as:

1. The mean $\mu(t)= E(\gamma(t))$
2. The variance $\sigma^2 (t) = Var(y(t)) = \gamma(0)$

There are two kinds of stationary:

2.3.1 on-Stationary around Variance

In the case fluctuation of time series about the contrast and this discrepancy is not fixed, it means that the series is stationary about the contrast, and there are transfers to convert the string non stationary to a series of stationary, including the conversion logarithmic and transfers of power and the square root of the absence of stationary, about the contrast non-fixed and turn it into a series of fixed and stationary contrast by applying the following formula [16]:

$$X_T = \begin{cases} X_t^{\lambda} & \lambda \neq 0 \\ X_t & \lambda = 0 \end{cases} \qquad \text{………………. (2.2)}$$

Where: X_t: the original series

2.3.2 Non-Stationary around the Mean

The basic conditional in being a stationary time series about mean and middle hard as the changes that occur in the qualities and characteristics of chains with time makes it unstable so you must remove the property not stability, of these chains are used difference method (Difference) to convert the string unstable to a series stable in terms of time difference and take her first and be in the following format [16]:

$$\Delta X = X_{t_t} - X_{t-1}$$

………………. (2.3)

$$W_t = \Delta X_t = X_t - X_{t-1}$$

………………. (2.4)

Where:

Δ: the difference factor

W_t: the new series

X_t: the original series

2.4 Box-Jenkins Models:

This is a methodology that George-Box and Gwilyn Jenkins at 1970 applied to time series data. Box and Jenkins popularized an approach that combines the moving average and the autoregressive approaches. A Box-Jenkins model explains that the time series is stationary or not. Box and Jenkins is recommended that the non-stationary differencing one or more times series to obtain stationarity, with

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the "I" standing for "Integrated" of an ARIMA model. A Box-Jenkins methodology is a powerful approach to the solution of many time series analysis problems. This methodology depends on parts of procedure which is [autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA)] that can be explained as follow:

2.4.1 Autoregressive (AR) model

The order of this type model depends on the of the significant partial autocorrelation function (PACF), its order is denoted by (P), the AR model can be written as follow:

$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + a_t$ ……………. (2.5)

By using backshift operator equation (2.6) can be rewrite as follow:

$\phi_p(B_p) X_t = a_t$ …………………. (2.6)

Where: $\phi_p(B_p) = (1 - \phi_1 B_1 - \ldots - \phi_p B^p)$

X_t : is the origin series.

a_t : is white noise, $a_t \sim N(0, \sigma_a^2)$

ϕ_p : is the estimated PACF.

To find Variance-Covariance the equation (2.6) should be multiplied by (X_{t-k}) and taking expectation so we get:

$E(X_t X_{t-k}) = E(\phi_1 X_{t-1} X_{t-k} + \phi_2 X_{t-2} X_{t-k} + \ldots + \phi_p X_{t-p} X_{t-k} + a_t X_{t-k})$ ..…(2.7)

Note:

$E(X_t X_{t-k}) = \lambda_k$.

$E(a_t X_{t-k}) = 0$.

Then:

$\lambda_k = \phi_1 \lambda_{k-1} + \phi_2 \lambda_{k-2} + \ldots + \phi_p \lambda_{k-p}$; $K > 0$ …..…………(2.8)

To get the ACF the equation (2.8) should be divided by the variance of the series (Υ_0).

$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \ldots + \phi_p \rho_{k-p}$ ……………. (2.9)

Note:

$\frac{\lambda_k}{\lambda_o} = \rho_k, \quad \Upsilon_0 = \sigma_X^2$

Then the PACF for the AR(P) model can be estimated by using Yule-Walker equations

$\rho_j = \phi_k \rho_{j-1} + \phi_{k(k-1)} \rho_{j-2} + \ldots + \phi_{kk} \rho_{j-p}$ ……………. (2.10)

2.4.2 Moving Average (MA) Model:

The order of moving average model depends on the number of significant ACF and $-\Theta_1$ is the coefficient of dependency of observations (Xt) on the error term et and the previous error term at-1, the MA(q) model can be write as follow

$X_t = a_t - \Theta_1 a_{t-1} - \Theta_2 a_{t-2} - \ldots - \Theta_q a_{t-q}$ ……………. (2.11)

Equation (2.11) can be rewrite with back shift operator as follow:

$X_t = \Theta(B) a_t$ …………………. (2.12)

Where:

$\Theta(B) = 1 - \Theta_1 B - \ldots - \Theta_q B^q$

The Var-Cov of MA(q) model is:

$$\Upsilon k = \begin{cases} \sigma_a^2 (-\Theta + \Theta_1 \Theta_{k+1} + \ldots + \Theta_q \Theta_{q-k}) & ; K = 1, 2, \ldots, q \\ 0 & ; K > q \end{cases}$$ …….(2.13)

$\Upsilon_0 = \sigma_a^2 \sum_{i=a}^{q} \Theta_i^2$ …………………... (2.14)

Note:

$\Theta_0 = 1$.

And the ACF is:

$$Pk = \begin{cases} \frac{-\Theta_k + \Theta_1 \Theta_{k+1} + \ldots + \Theta_q \Theta_{q-k}}{1 + \Theta_1^2 + \Theta_2^2 + \cdots + \Theta_q^2} & ; k = 1, 2, \ldots, q \\ 0 & ; k > q \end{cases}$$ ………... (2.15)

2.4.3 Autoregressive Moving Average Model (ARMA)

There is large family of models which is named "Autoregressive-Moving Average Models" and abbreviated by ARMA. Many of researchers in different application fields prove that ARMA models fits more than other traditional methods for forecasting. The ARMA model is a more general model as a mixture of the AR(p) and MA(q) models and it is called an autoregressive moving average

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model (ARMA) of order (p,q). The ARMA(p,q) is given by:

$$\phi_p(B)X_t = \odot_q(B)a_t \quad \dots\dots\dots\dots\dots\dots (2.16)$$

Where:

$$\phi_p(B) = 1 - \phi_1 B - \dots - \phi_p B^p$$
$$\odot_q(B) = 1 - \odot_1 B - \dots - \odot_q B^q$$

We write equation (2.16) as:

$$X_t = \phi_1 X_{t-1} + \dots + \phi_p X_{t-p} + a_t - \odot_1 a_{t-1} - \dots - \odot_q a_{t-q} \dots (2.17)$$

2.4.4 The Autoregressive Integrated Moving Average Models (ARIMA)

When the time series data is not stationary, difference operator can be used to remove non-stationary, the time series data after differencing is called adjusted data and the fitted model is called integrated model which is combine both autoregressive and moving average models.

Notationally, all AR(p) and MA(q) models can be figured as ARIMA(1, 0, 0) this tells there is no differencing and no MA part. The general of ARIMA is written as ARIMA (p, d, q) where p is the order of the AR part, d is the degree of differencing and q is the order of the MA part , [16].

$$W_t = \nabla^d X_t = (1 - B)dX_t$$

The general ARIMA process is of the form

$$X_t = \sum_{t=1}^{p} \phi_i X_{t-1} + \sum_{t=1}^{q} \odot_i a_{t-1} + \mu + a_t \quad \dots\dots\dots\dots\dots\dots (2.18)$$

2.5 Autocorrelation Function (ACF)

The autocorrelation function measures the degree of correlation between neighboring observations in a time series. The autocorrelation coefficient is estimated from sample observation using the formula :

$$p_k = \frac{\sum_{r-2}^{n}(X_t - \mu_x)(X_{t+k} - \mu_x)}{\sum_{r=1}^{n}(X_t - \mu_x)^2} \dots\dots\dots\dots\dots\dots (2.19)$$

Thus, the autocorrelation function at lag k is defined as:

$$p_k = \frac{\lambda_k}{\lambda_0}, \quad k = 0, \pm 1, \pm 2, \dots$$

2.6 Partial Autocorrelation Function (PACF)

The partial autocorrelation function at lag k is the correlation between X_t and X_{t-k} after removing the effect of the intervening variables X_{t-1}, X_{t-2}, ….., X_{t-k+1} which locate within (t, t-k) period, partial autocorrelation function will be donated by ϕκκ, PACF is calculated by iteration , [21].

$$\phi_{00} = 1$$
$$\phi_{11} = p_1$$
$$\phi_{kk} = \frac{p_{kk} - \sum_{j=1}^{k-1} \phi_{k-1,j} p_{k-j}}{1 - \sum_{j=1}^{k-1} \phi_{k-1,j} p_j}, k = 2,3, \dots \quad \dots\dots\dots\dots\dots\dots (2.20)$$

Therefore $\phi_{kj} = \phi_{k-1,j} - \phi_{kk}\phi_{k-1,k-1}, \quad j = 1,2,\dots, k-1$

2.7 Model Selection Criteria

2.7.1 Akaike Information Criterion AIC

Akaike Information Criterion AIC is defined as

$$AIC = -2 \log L + 2p \quad \dots\dots\dots\dots\dots\dots (2.21)$$

where L is the maximized likelihood function and p is the number of effective parameters. The best model is the one with the smallest AIC. The likelihood function part reflects the goodness of fit of the model to the data, while 2p is described as a penalty. Since L generally increases with p, AIC reaches the minimum at a certain p. AIC is based on the information theory.

2.7.2 Mean Absolute Percentage Error (MAPE)

The MAPE also is said to be mean absolute percentage deviation (MAPD) that is an accurate measure of a method for constructing time series fitted model, the accuracy in time series processes is expressed by:

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$$M = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{X_t - \hat{X}_t}{X_t}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots.(2.22)$$

Where X_t is the actual value and $\hat{X}_t$ is the forecast value.

2.7.3 Mean Absolute Error (MAE)

The MAE is mathematically expressed by:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|e_t| \quad \ldots\ldots\ldots\ldots\ldots\ldots.(2.23)$$

Where *it* is the error term and n is the number of forecasting .

2.7.4 Root Mean Square Error (RMSE)

The RMSE is expressed by:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(X_i - \hat{X}_i\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots.(2.24)$$

Where: X_i = actual value

$\hat{X}_i$= forecasteed value

N = number of forecasted time period [16].

2.8 Estimating the Parameters of an ARMA Model

Iterative method can be used to estimate the parameters of the ARMA model. At every point sum square residual should be calculated of suitable grid of the parameter values, and the sufficient values are given minimum sum of squared residuals. For an ARMA (1,1) the model is given by

$X_t - \mu = \phi_1(X_{t-1} - \mu)a_t + \odot_1 a_{t-1}$ ………………. (2.25)

Given N observation $X_1, X_2, \ldots, X_N$, we guess values for μ, ϕ_1, $\odot_1$, set $a_0 = 0$ and $Y_0 = 0$ and then calculate the residuals recursively by

$a_1 = X_1 - \mu$

$a_2 = X_2 - \mu - \phi_1(X_1 - \mu) - \odot_1$

$a_N = X_N - \mu - \phi_1(X_1 - \mu) - \odot_1 a_{N-1}$

The residual sum of squares $\sum_{t=1}^{N} a_t^2$ is calculated. Then other values of μ, ϕ_1, $\odot_1$, are tried until the minimum residual sum of squares is found.

Note: It has been found that most of the stationary time series occurring in practices can be fitted by AR(1), AR(2), MA(1), MA(2), ARMA(1,1) or white noise models that are customarily needed in practice.

2.9 Models Forecasts

The main goal of constructing a model for a time series is to make future forecasteions for a given series. It also plays a significant role in assessing the forecasts accuracy. The ultimate test of an ARIMA model is power or ability to forecast. In order to obtain a forecast with a minimal errors, there are seven features of a good ARIMA models taken into account. First, a good model is parsimonious. That is, it has the smallest number of coefficients which explain the data set. Secondly, a sufficient AR model should not be nonstationary. Thirdly, the MA of the model should be invertible. Fourth, insufficient model the residuals must be independent. Fifth, the distribution of residuals of a good model must be distributed normal . From the existing theory of the series up to time t, namely, $X_1, X_2, X_3, \ldots, X_{t-1}, X_t$, we can forecast the value of X_{t+h} **,** that will happen h time units ahead. In this case, time t is the forecast origin and the lead time forecast. This forecast is denoted and estimated as

$$\hat{X}_t(L) = E(X_{t+h}|X_1, X_2, \ldots\ldots, X_t)\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.26)$$

Once an adequate and satisfactory model is fitted to the series of interest, forecasts can be generated using the model.

$x_t = \phi_1 x_{t-1} + \cdots + \phi_p x_{t-p} + a_1 - \odot_1 a_{t-1} \ldots - \odot_q a_{t-q}$………… (2.27)

The one-step-ahead forecast for time $t + 1$ is given by:

$x_{t+1} = \phi_1 x_t + \cdots + \phi_p x_{t-p+1} + a_{t+1} - \odot_1 a_t \ldots - \odot_q a_{t-q+1}$…. (2.28)

2.10 Haar Wavelet

2.10.1 Haar Wavelet

The simplest type of wavelet is the Haar wavelet. In its discrete form, these wavelets are linked with a mathematical process that is known as the Haar transform. This process plays the role of a prototype that facilitates other wavelet transforms. If we want to get a good understanding of the sophisticated wavelet transforms, we need to gather more information about the Haar transform. In short, this wavelet transform can be considered as the most suitable choice regarding localized jumps and edge detection [24]. The numeric definition for the Haar scaling function is as follows:

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$$\phi(x) = \begin{cases} 1, & if\ 0 \le x < 1 \\ 0, & otherwise. \end{cases}$$

Whereas, the definition for the Haar mother wavelet is as follows:

$$\psi(x) = \phi(2x) - \phi(2x-1) \qquad \text{............ (2.29)}$$

$$\psi(x) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x < 1 \\ 0, & otherwise. \end{cases}$$

2.10.2 Haar Wavelet's properties

- Any function can be used as the constant function's linear combination, $\psi(x), \psi(2x), \psi(2^2x), \ldots, \psi(2^kx), \ldots$ and their shifting functions.
- Another property is that any function can be used as the linear combination of $\phi(x), \phi(x2), \phi(2^2x), \ldots, \phi(2^kx), \ldots$ and associated shifting functions.
- Another property is that only Haar Wavelet can be compactly supported orthogonal and has symmetry
- Last property is that it makes use of a set of functions $\{2^{\frac{j}{2}}\phi(2^j\,x - k); k \in Z\}$ on orthonormal basis [24].

2.10.3 Level Haar Transform

1st level Haar Transform for $f = (x_1, x_2, x_3, \ldots, x_N)$ is given by:

$$f \xrightarrow{H1} (a^1|d^1) \qquad \text{............ (2.30)}$$

$$a^1 = \frac{x_1+x_2}{\sqrt{2}}, \frac{x_3+x_4}{\sqrt{2}}, \ldots, \frac{x_{N-1}+x_N}{\sqrt{2}} \qquad \text{............ (2.31)}$$

$$d^1 = \frac{x_1-x_2}{\sqrt{2}}, \frac{x_3-x_4}{\sqrt{2}}, \ldots, \frac{x_{N-1}-x_N}{\sqrt{2}} \qquad \text{............ (2.32)}$$

The list continues so for other levels [24].

2.10.4 Advantages of the Haar Wavelet Transform

It is to be mentioned that the Haar Wavelet Transform offers multiple advantages. Some of the main advantages of the Haar Wavelet Transform include:

- conceptually simple,
- fastest possible wavelet,
- fast processing speed,
- reversibility, without the edge effects that are a problem with other Wavelet transforms,
- increased memory efficiency, since it can be calculated in place without a temporary Array [24]

2.10.5 The Haar Transform Limitations

As for the limitations of the Haar Transform, which can be a problem with for some applications? In generating each of averages for the next level and each set of coefficients, the Haar transform performs an average and difference on a pair of values. Not only this, when generating the averages to be used at the next level as well as for each set of coefficients, this transform performs with regard to the pair of values in which the algorithm makes a shift over by two consecutive values for calculating the difference level. Moreover, the Haar transform window is only wide by two elements so if in case a big change occurs from an even to an odd value, we cannot see the change on the high frequency coefficients. Therefore, it can be said that for audio signal compressing and for noise removal, the Haar wavelet transform cannot be a viable choice [24].

3. Data Analysis and Results

3.1 Data Description

The dataset used for the analysis in this study is contained one variable and deals with yearly silver closing price since January 1969 up to 2022 as shown in figure 4.1.

TABLE 3.1
DATA DESCRIPTIONS

Mean	10.17907
Median	6.330000
Maximum	35.12000
Minimum	1.540000
Std. Dev.	7.912488
Skewness	1.233229
Kurtosis	3.865170
Jarque-Bera	15.37184
Probability	0.000459
Sum	549.6700
Sum Sq. Dev.	3318.196
Observations	54

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3.2 Applications

The time series plots are display observations on the y-axis against equally spaced time intervals on the x-axis. They are used to evaluate patterns, knowledge of the general trend and behaviors in data over time. The time series plot of m yearly silver closing price is displayed in Figure 3.1 below:

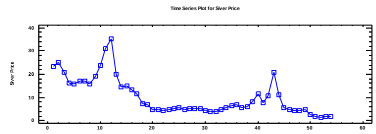

Figure 3.1: yearly plot of time series of closing silver price

Figure 3.1 indicates that the data of time series is not random. The plot shows consistent pattern of short-term changes for data which indicates the existence of trend fluctuations. This series varies randomly over time and there are trend fluctuations. For further testing of the stationery of the time series, we applied Box-Pierce Test for yearly closing silver price, and demonstrates this result by the examination of the autocorrelation and partial autocorrelation functions as shown below.

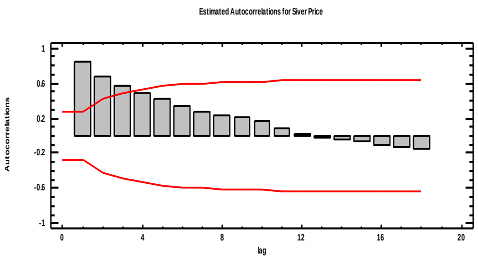

Figure 3.2: Autocorrelation Function for yearly closing silver price

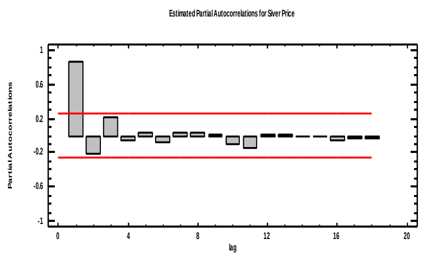

Figure 3.3: Partial Autocorrelation Function for yearly closing silver price

All the above results and plots support that the data of time series is not random at the level.

TABLE 3.2
BOX-PIERCE TEST FOR LEVEL OF SERIES

Randomness Test	Value	P-value
Box-Pierce Test	125.018	0.000

Since the P-value for the test in table (3.2) is less than 0.05, we can reject the hypothesis that the series is random at the 95.0% confidence level. Since the three tests are sensitive to different types of departures from random behavior, failure to pass any test suggests that the time series may not be completely random. And it needs some treatments to be transformed to a random series. Therefore, we used many transformations and we found that the most suitable transformation is by differencing the series. We note that the time series for the first-differenced series in Figure 3.4.

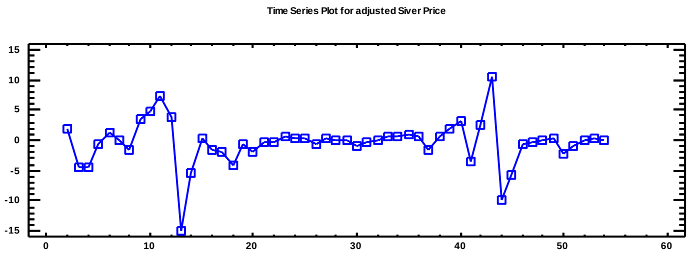

Figure 3.4: Time series plot of the first difference of yearly closing silver price

Figure 3.4 indicates that the data of time series at first difference which is not random. The plot shows consistent pattern of short-term changes for data, and this demonstrates by estimating the autocorrelation and partial autocorrelation function (ACF and PACF) for the first-differenced series in Figure 3.5 and 3.6

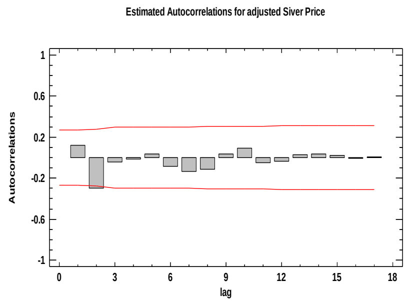

Figure 3.5: Autocorrelation Function for the first-differenced series of yearly

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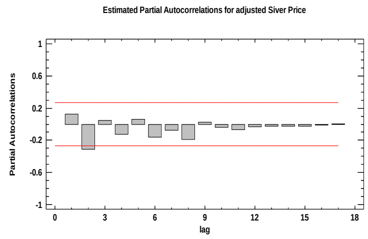


closing silver price

Figure 3.6: Partial Autocorrelation Function for the first-differenced series of yearly closing silver price

The results above demonstrate the first differencing of the time series data of yearly closing silver price. Thus, the series became is not stationarity.

TABLE 3.3
BOX-PIERCE TEST FOR FIRST DIFFERENCING OF SERIES

Randomness Test	Value	P-value
Box-Pierce Test	8.4604	0.000

The Box-Pierce Test in table (3.3) is based on the sum of squares of the first 24 autocorrelation coefficients. Since the P-value for this test is greater than or equal to 0.05, we cannot reject the hypothesis that the series is random at the 95.0% or higher confidence level. Since the three tests are sensitive to different types of departures from random behavior, failure to pass any test suggests that the time series may not be completely random. And it needs second differencing of the series in Figure 3.7.

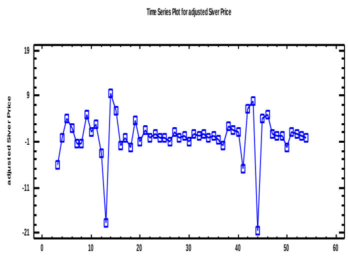
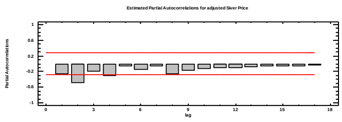


Figure 3.7: Time series plot of the second difference of yearly closing silver price

Figure 3.7 indicates that the data of time series at second difference which is random. The plot shows consistent pattern of short-term changes for data, and this demonstrates by estimating the autocorrelation and partial autocorrelation function



(ACF and PACF) for the first-differenced series in Figure 3.8 and 3.9

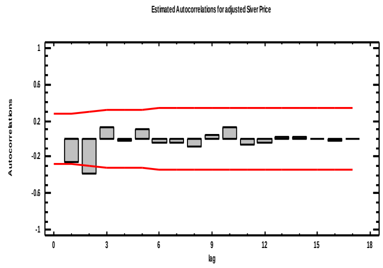


Figure 3.8: Autocorrelation Function for the second-differenced series of yearly closing silver price

Figure 3.9: Partial Autocorrelation Function for the second-differenced series of yearly closing silver price

The results above demonstrate the first differencing of the time series data of yearly closing silver price. Thus, the series became is not stationarity.

TABLE 3.4
BOX-PIERCE TEST FOR SECOND DIFFERENCING OF SERIES

Randomness Test	Value	P-value
Box-Pierce Test	13.8657	0.6765

The Box-Pierce Test in table (3.4) is based on the sum of squares of the first 24 autocorrelation coefficients. Since the P-value for this test is greater than or equal to 0.05, we cannot reject the hypothesis that the series is random at the 95.0% or higher confidence level.

3.4 Model Identification

This section shows how we determine the order of the ARIMA model. We computed all relevant criteria to determine good ARIMA model of yearly closing silver price. Those are the ACF and PACF in addition to RMSE, MAE, MAPE, and ME criteria. To take a decision must be scanning all the plots of ACF and PACF coefficients of the series as shown in the figure (3.8 and 3.9) respectively. yearly silver closing price data is yearly and according to the identification criteria, the following models have been examined and estimated as shown in table (3.5) below. The best model is chosen through the RMSE,

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MAE, MAPE and ME criteria if it shows the lowest values of these criteria as it is shown in table (3.5).

TABLE 3.5
ARIMA MODELS CRITERIA FOR YEARLY CLOSING SILVER PRICE

Model	RMSE	MAE	MAPE	ME
ARIMA(1,2,1)	3.57631	2.16301	26.7392	-0.34888
ARIMA(1,2,2)	3.59263	2.18821	20.8126	0.158399
ARIMA(2,1,0)	3.53923	2.13005	21.5809	-0.46186
ARIMA(2,2,0)	3.58987	2.0965	20.2744	0.187463
ARIMA(2,2,1)	3.5354	2.0727	20.6897	0.085184

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The initial order of the ARIMA model have been determined from the figure (3.8 and 3.9) for MA and AR respectively, from figure 3.8 we have only one significant ACF, while from figure 3.9 we have two significant PACF, thus the model is ARIMA (2, 2, 1), and it is shown in table (3.5) that the ARIMA (2, 2, 1) model produced the value of each RMSE, MAE, and ME criteria with the smallest values. This means that the ARIMA (2, 2, 1) model is the best among all the other models, which is the most suitable model that can be obtained for yearly closing silver price.

3.5 Parameters Estimation

Since we concluded in the previous section that the ARIMA (2,2,1) model is the best model with the smallest value of RMSE, MAE, MAPE and ME criteria, the parameters had been estimated using the method of maximum likelihood estimation as it is the best and most appropriate method of estimation. The results of the parameters estimation of the model are shown in table (3.6) below.

TABLE 3.6
PARAMETER ESTIMATES OF ARIMA (2,2,1) MODEL ESTIMATE MODEL COEFFICIENTS

Parameter	Estimate	Stnd. Error	t	P-value
AR(1)	0.185715	0.1365	1.36056	0.180009
AR(2)	-0.324387	0.136853	-2.37034	0.021837
MA(1)	1.03162	0.0469788	21.9593	0.000000

It is shown in table (3.6) that the p-value for the parameters AR(2), and MA(1) coefficients are less than $\alpha = 0.05$. This indicates that these coefficients are significantly different from zero, however the p-value of AR(1) parameter is greater than $\alpha = 0.05$, means that this coefficient is not significantly different from zero. As it is shown for this model, the RMSE, MAPE, MAE and AIC criteria are the smallest values among the other models. Thus, the final model is ARIMA (2,2,1).

3.7 Applying a wavelet Transformation

In this section applying a wavelet residual modification to improve prediction precision by ARIMA(2, 2, 1) model the first step is make a residual which are obtained from the model as shown in the table (3.7)

The second step is final prediction value as shown in table (3.7).

TABLE 3.7
RESIDUAL OBTAINED FROM ARIMA(2, 2, 1)

Period	Residual	Period	Residual	Period	Residual
1		19	0.175433	37	-0.869712
2		20	-2.41981	38	1.49418
3	-3.48998	21	0.355792	39	1.80695
4	-2.09139	22	-0.250384	40	3.55612
5	-0.256499	23	1.0067	41	-2.91471
6	0.841174	24	0.572026	42	4.70361
7	0.522762	25	0.938143	43	9.42665
8	-0.252398	26	-0.124333	44	-10.1994
9	4.5276	27	0.983946	45	0.0735264
10	4.52249	28	0.20678	46	-2.48509
11	8.63589	29	0.641661	47	-1.61851
12	5.43132	30	-0.544375	48	0.166785
13	-11.8333	31	0.264027	49	0.399999
14	-0.409394	32	0.257843	50	-1.87934
15	-2.48313	33	1.01753	51	-0.22087
16	-2.54604	34	0.957304	52	-0.517847
17	-0.61599	35	1.55795	53	0.0833951
18	-3.65297	36	0.945617	54	0.0318247

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TABLE 3.8
SHOWS ACTUAL VALUES, ARIMA (2, 2, 1) VALUES, WAVELET FORECAST WITH WAVELET RESIDUAL

Period	Data	Forecast	Wavelet Forecast	Wavelet Residual
1	23.3			
2	25.1			
3	20.7	24.1800	23.3494	-2.6594
4	16.2	18.3114	17.8136	-1.5936
5	15.7	15.9665	15.9055	-0.1955
6	17.1	16.2288	16.4290	0.6410
7	17.2	16.6472	16.7716	0.3984
8	15.7	15.9124	15.8523	-0.1923
9	19.1	14.5424	15.6200	3.4500
10	23.8	19.2675	20.3439	3.4461
11	31.2	22.5141	24.5694	6.5806
12	35.1	29.6887	30.9814	4.1386
13	20.2	32.0233	29.2070	-9.0170
14	14.7	15.0794	14.9820	-0.3120
15	15	17.4731	16.8821	-1.8921
16	13.4	15.9260	15.3200	-1.9400
17	11.6	12.1660	12.0194	-0.4694
18	7.31	10.9630	10.0936	-2.7836
19	6.66	6.4846	6.5263	0.1337
20	4.88	7.2998	6.7239	-1.8439
21	4.6	4.2442	4.3289	0.2711
22	4.37	4.6204	4.5608	-0.1908
23	4.95	3.9433	4.1829	0.7671
24	5.22	4.6480	4.7841	0.4359
25	5.54	4.6019	4.8251	0.7149
26	4.9	5.0243	4.9947	-0.0947
27	5.2	4.2161	4.4502	0.7498
28	5.2	4.9932	5.0424	0.1576
29	5.29	4.6483	4.8011	0.4889
30	4.31	4.8544	4.7248	-0.4148
31	3.95	3.6860	3.7488	0.2012
32	4.06	3.8022	3.8635	0.1965
33	4.83	3.8125	4.0546	0.7754
34	5.5	4.5427	4.7705	0.7295
35	6.53	4.9721	5.3428	1.1872
36	7.02	6.0744	6.2994	0.7206
37	5.47	6.3397	6.1327	-0.6627
38	6.13	4.6358	4.9914	1.1386
39	8.15	6.3431	6.7731	1.3769
40	11.4	7.8639	8.7102	2.7098
41	7.92	10.8347	10.1410	-2.2210
42	10.5	5.7864	6.9058	3.5842
43	21	11.5534	13.7969	7.1831
44	11.1	21.2694	18.8419	-7.7719
45	5.42	5.3465	5.3640	0.0560
46	4.64	7.1251	6.5336	-1.8936
47	4.35	5.9685	5.5833	-1.2333
48	4.43	4.2632	4.3029	0.1271
49	4.67	4.2700	4.3652	0.3048
50	2.55	4.4293	3.9821	-1.4321
51	1.68	1.9009	1.8483	-0.1683
52	1.54	2.0579	1.9346	-0.3946
53	1.77	1.6866	1.7064	0.0636
54	1.8	1.7682	1.7758	0.0242

Table (3.8) shows the real value and forecasting precision obtained from wavelet, then one can test the accuracy of the models according to mean error (ME) and root mean square error (RMSE) which shown in the table (3.9)

TABLE 3.9
MODEL ACCURACY

Measurements	ARIMA(2,2,1)	Wavelet ARIMA(2,2,1)
RMSE	3.463972	2.639544239
ME	0.085184	0.064910685

Table 3.9 shows the comparison between ARIMA(2,2,1) and wavelet ARIMA(2,2,1) which is depends on ME and RMSE, it is clear that the ARIMA(2,2,1) after transforming to the wavelet is the best accurate than the model before using wavelet transformation.

5. Conclusion

Silver is a precious metal and the spot price not only reflects the current supply and demand condition but it also reflects investors' expectations of future inflation. Therefore, the study aims to analyze the time series of yearly silver closing price for the period between (1969 to 2022) using time series analysis which is (Box-Jenkins) method for the accuracy and flexibility it has in addition modifying the prediction by using wavelet transformation. The results of the study shown that the wavelet ARIMA(2,2,1) model has a great effect on the accuracy of the model, the root mean square error is only 2.6395 of the modification model, and forecasting accuracy is greatly improved and better than the model before modification, and the statistical tests show that the time series of yearly silver closing price is stable, WHILE the best and most efficient model is ARIMA (2,2,1) among the possible models which was chosen using the balancing standards (the smallest value of each: AIC, RMSE, MAPE and MAE criteria). The main result of the study is the parameters estimate of ARIMA (2,2,1) Model which are significant thus, the ARIMA (2,2,1) is efficient. Finally, the future study can be use wavelet GARCH model to improve and determine volatility in silver closing price.

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